A Strong Constraint on Ever-Present Lambda

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Abstract

We show that the causal set approach to creating an ever-present cosmological 'constant' in the expanding universe is strongly constrained by the isotropy of the microwave background. Fluctuations generated by stochastic lambda generation which are consistent with COBE and WMAP observations are far too small to dominate the expansion dynamics at $z < 1000$ and so cannot explain the observed late-time acceleration of the universe. We also discuss other observational constraints from the power spectrum of galaxy clustering and show that the theoretical possibility of ever-present lambda arises only in 3+1 dimensional space-times.

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The apparent existence of a non-zero cosmological constant, Λ, with a positive value of order 10^{-120} in Planck units is a mystery to astronomers and a challenge to the ingenuity of theoretical physicists. It may be explained by some unknown, or as yet partially known, fundamental theory of everything which prescribes the vacuum state of the universe uniquely and completely. Equally, it might arise through a random symmetry-breaking process within a landscape containing a huge (or even an infinite) number of different possible vacua. In the latter case, it would make no more sense to try to predict the observed value of Λ from the underlying theory than to use a theory of dynamics to predict how many planets there should be in the Solar System. All a theory of Λ could do is to assess the likelihood of the observed value within the subset of outcomes that permit the evolution of 'observers'. As first shown in ref [1], the constraint on Λ from the requirement that galaxy and star formation be possible places an upper bound on the allowed magnitude of Λ that we (or other 'observers') could observe that is only about an order of magnitude weaker than the observed value. The observed value of Λ [2] implies that the universe has accelerated for the last 25% of its expansion scale-factor history, and the energy densities contributed by Λ and by the other material stresses in the Friedmann equation governing the expansion of the universe are still of comparable order. Thus, there are two puzzles about Λ: why is it non-zero and why did it become dynamically significant so close to the present epoch?

One specific attempt to address these questions with a simple testable model is that proposed by Ahmed et al [3]. It notes that the magnitude of the observed

Λ in Planck units is of order $N^{-1/2}$, where N is the spacetime volume of the universe (Hubble volume $\times$ expansion age $\propto t^4$) in Planck units. Thus, at comoving proper expansion time t, we have $N(t) \sim (t/t_p)^4$ and $\Lambda \sim N^{-1/2} \sim (t_p/t)^2$, where the Planck time is $t_p = (G\hbar/c^5)^{1/2} \sim 10^{-43}s$. Therefore, today, at $t_0 \sim 10^{17}s$, we should observe Λ as a residual quantum gravity effect with a magnitude

$$\Lambda \sim (N(t_0))^{-1/2} \sim (t_p/t_0)^2 \sim 10^{-120}. \tag{1}$$

In this model the induced value of Λ at any time t will always be of order $(t_p/t)^2$, and so is always of the same order as both the dominant matter or radiation density and the square of the Hubble expansion rate H^2 in the Friedmann equation. It is for this reason that a Λ stress originating in this fashion has been called 'ever-present lambda'.

We note that this type of 'ever-present' scenario is only a possibility in a universe with three space dimensions. If the universe has D spatial dimensions then the spacetime volume grows as t^{D+1}, and the magnitude of the lambda density induced by Poisson fluctuations in the number of Planck-sized regions in the whole $(D+1)-$dimensional spacetime volume is $\rho_\Lambda \propto t^{-(D+1)/2}$. However, the Hubble parameter and the dominant matter (or radiation) density still evolve at $H^2 \sim \rho \sim t^{-2}$ and so are only always of comparable order to ρ_Λ for worlds with $D = 3$. If $D > 3$, then ρ_Λ falls off faster than t^{-2} and is never $O(\rho)$ at late times; whereas, if $D < 3$, ρ_Λ dominates ρ at late times.

Ahmed et al [3] propose a specific mechanism by which the induced Λ term can arise as a Poisson fluctuation in $N(t)$ by means of a causal-set description of the geometry of four-dimensional spacetime, which is reviewed in ref [3, 4]. Fluctuations arise as the number of causally connected Hubble four-volume grows with the expansion of the universe to encompass more Planck-sized spacetime volumes [3]. The associated quantum uncertainty principle is $\delta\Lambda\delta N \gtrsim 1$, with $\delta N \sim N^{1/2}$. Specifically, the induced Λ is assumed to arise from a Poisson process on the number of independent Planck-sized spacetime volumes contained within the particle horizon at time t. If at the n^{th} timestep, t_n, this number of Planck four-volumes is N_n, we can define its change with respect to the number of Planck-sized spacetime volumes in the horizon at time t_n, given by $V(t_n)$, by the difference

$$\delta N_n \equiv N_{n+1} - N_n = V(t_{n+1}) - V(t_n). \tag{2}$$

Hence, at the $(n+1)^{st}$ timestep, the induced Λ will be taken to contribute an energy density to the Friedmann equation equal to

$$\rho_{\Lambda,n+1} = \frac{S_{n+1}}{N_{n+1}} = \frac{S_n + \alpha\xi_{n+1}\sqrt{\delta N_n}}{N_n + \delta N_n} \sim \frac{\alpha\xi_{n+1}}{\sqrt{N_{n+1}}}, \tag{3}$$

where S_i are the sums of the first i random numbers with $S_{i=0} = 0$. The Poisson process is assumed to generate random numbers ξ_i with zero mean and standard deviation 1 and the fluctuation intensity is controlled by the free constant α.

The contribution of the induced ρ_Λ to the Friedmann equation takes the usual form:

$$3H^2 = \rho_m + \rho_\gamma + \rho_{\Lambda}, \tag{4}$$

where ρ_m and ρ_γ are the densities of matter and radiation. Numerical simulations [3] confirmed that the induced ρ_Λ term is always similar in magnitude to the largest of ρ_m and ρ_γ during the evolution of the universe if α is sufficiently large. There are some technical issues arising from this model: it will have to be cut off at the extremes to stop very large negative contributions to Λ being inconsistent with the positivity of the left-hand side of (4), (note that 50% of the time the induced lambda fluctuations are negative) and the quantum physics of the causal-set generation of the fluctuations within the context of some unimodular theory of gravity remains to be explored in detail [4]. The occurrence of large negative fluctuations can be rendered innocuously improbable over the life of the universe by choosing α to be sufficiently small and we shall ignore these difficulties. The whole model could be rendered more rigorous by adopting to a full stochastic differential equation formulation. However, we do not intend to investigate these aspects of the scenario here. Rather, we are interested in the best observational bounds on the allowed value of the undetermined statistical intensity parameter, α, which is the one free parameter in the model.

In their study, Ahmed et al [3] consider three principal constraints on α: α must not be too small if we want the universe to accelerate in the recent past because we need the induced lambda fluctuations to permit $\rho_\Lambda \gtrsim \rho_m$ today, as observed [2], with reasonable probability; α must not be too large, or large negative lambda fluctuations will be too probable and contradict $H^2 > 0$ in eqn. (4); α must not be so large as to create changes in the expansion rate of the universe at redshift $z \sim 10^{10}$ that lead to big bang nucleosynthesis of unacceptable abundances of helium-4 with significant likelihood. There is some tension between these opposing requirements but the best compromise appears to be for the range with $\alpha = 0.01 - 0.02$. It might also be the case that an alternative formulation could require the lambda fluctuations to be positive semi-definite.

There is another constraint on the scenario that might be expected to produce an upper bound of $\alpha \lesssim 0.02$ or better. The induced lambda contributions are of order α times the dominant density that drives the expansion. At cosmological times close to the epoch of matter-radiation equality, at redshift $1 + z_{eq} \sim 2.4 \times 10^4$, there will be small corrections to the equal-density time in the ever-present lambda model that can be determined accurately by a simulation. However, they must be smaller than about 2% in amplitude or they will shift the epoch of equal density sufficiently to move the peak of the power spectrum of galaxy clustering away from its observed position with significant probability. This is a similar bound to that used by Tegmark [5] to constrain light neutrino masses, and Liddle et al [6] to constrain Brans-Dicke theories. It arises because density perturbations in the matter only commence growth by gravitational instability after the radiation-dominated era ends.

In the earlier analysis of ref. [3], it was assumed that the stochastic process generating the lambda fluctuations is perfectly homogeneous and isotropic, so $\rho_\Lambda \equiv \rho_\Lambda(t)$. Thus, in effect, it simply transforms the dynamics along the line of exact Friedmann universes containing matter, radiation and instantaneous lambda-like stresses. We believe that this is not a physically complete representation of the underlying stochastic process. Rather, we would expect the fluctuation to be at best only statistically homogeneous and isotropic because of its quantum gravitational origin. As a result of quantum uncertainty, there will always be small statistical fluctuations in the overall homogeneity and isotropy of the expansion dynamics on the horizon scale, yielding $\rho_\Lambda \equiv \rho_\Lambda(\mathbf{x}, t)$. These will manifest themselves as small gravitational potential fluctuations on the horizon scale. When the universe cools sufficiently for atomic recombination to occur at the epoch of last scattering of the microwave background photons, at $z_{ls} \sim 1000$, the horizon-sized stochastic fluctuations will create temperature anisotropies in the microwave background of amplitude α. They will be equal in amplitude to the gravitational potential fluctuations produced by the density perturbations on a scale $L \sim t$, across angular scales of order a few degrees. These fluctuations contribute an amplitude to the observed microwave temperature anisotropy, $\Delta T/T$, that is a fraction α of the background density:

$$\Delta T/T \sim \Delta\Phi/\Phi \sim (\delta\rho_{_\Lambda}/(\rho + \rho_{_\Lambda}))(L/t)^2 \ \sim \alpha, \tag{5}$$

where $\Delta\Phi$ is the gravitational potential perturbation and ΔT is the angular anisotropy perturbation observed in the cosmic microwave background temperature. The observational data from the COBE [7] and WMAP [8] missions, along with complementary ground-based detections, therefore provide the powerful constraint:

$$\alpha \lesssim 10^{-5}. \tag{6}$$

This an extremely tight bound on the causal set mechanism for ever-present lambda generation and prevents any effective lambda generated by this mechanism from ever being large enough to dominate the dynamics at $0 \leq z < 1000$ with any significant probability, as is required to provide an explanation for the redshift-distance relation of type Ia supernovae at low redshifts [2]. This would require a more complicated stochastic generation model with an effective α that increased with time so that it could be $O(1)$ near the present but $O(10^{-5})$ at $t_{rec} \sim 10^{13}s$. We conclude that the simple Poisson model for causal-set generation of lambda fluctuations is constrained to produce an effectively never-present lambda relative to the cold dark matter density in the Friedmann equation.

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